Physics ATAR - Year 12

Gravity and Motion
Test 2
2019

Name: SOLUTIONS	Mark: / 61 = %

Time Allowed: 50 Minutes

Notes to Students:

1. You must include **all** working to be awarded full marks for a question.
2. Marks will be deducted for incorrect or absent units and answers stated to an incorrect number of significant figures.
3. **No** graphics calculators are permitted – scientific calculators only.

Question 1 **(6 marks)**

The top of Mount Everest is 8850 m above sea level.

(a) Calculate the effective magnitude of gravity on the top of Mount Everest. That is, the acceleration due to gravity of objects allowed to fall freely at this altitude.

(4 marks)

$r = r_E + 8500$
$= 6.37 \times 10^6 + 8850$
$= 6.378850 \times 10^6$ m (1)

$$g = \frac{GM_E}{r^2}$$ (1)

$$g = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.378850 \times 10^6)^2}$$ (1)

$= 9.79$ ms^{-2} (1)

*if students round $r = 6.38 \times 10^6$ m due to addition rule), allow $g = 9.78$ ms^{-2}.

(b) Provide two reasons why this value is considered an estimate.

(2 marks)

- Earth is not perfectly spherical / mean radius given in data sheet / Earth bulges at equator
- Earth is non-homogenous / not uniformly dense
- Have not taken into account centripetal acceleration due to Earth's rotation.

Question 2 **(5 marks)**

Charon orbits Pluto in 6.3873 Earth days, following a circular path with an average radius of 19,640 km. Calculate the mass of Pluto using the information given to 3 significant figures.

$\Sigma F = F_c = F_g$ (½) $v = \frac{2\pi r}{T}$ (½)

$$\frac{m_1 v^2}{r} = \frac{G m_1 m_2}{r^2}$$

$$\frac{m\frac{4\pi^2 r^2}{T^2}}{r} = \frac{G m 1 m_2}{r^2} \quad (1)$$

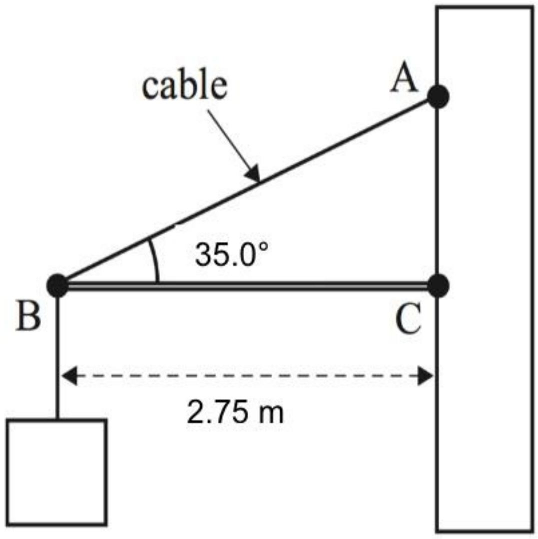
$$\frac{4 m_2 \pi r}{T^2} = \frac{G m_2}{r^2} \quad \rightarrow \quad m_2 = \frac{4\pi^2 r^3}{G T^2} \quad (1)$$

$$= \frac{4\pi^2 (19640\times10^3)^3}{(6.67\times10^{-11})(6.3873\times24\times60)^2} \quad (1)$$

$= 1.47 \times 10^{22}$ kg (1)

Question 3 **(9 marks)**

A 2.75 m horizontal beam of uniform mass is attached to a wall, as shown below. The mass of the beam is 25.0 kg and there is a hanging mass at point B. Point C is a hinge and the cable is attached an angle of 35.0° to the beam. The maximum tension the cable can provide before snapping is 1.75 kN.



(a) Calculate the maximum mass that can be suspended at point B if a safety factor of 3 is applied, that is, the maximum mass would cause a tension in the cable that is 3 times less than its snapping limit. (4 marks)

$\Sigma\tau = 0$ $\tau = rF\sin\theta$ (1)

$cwm = acwm$

Taking pivot about C (or set T = 583N)

2.75(mx9.8) + (2.75/2)(25x9.8) = 2.75($\frac{1750}{3}$)(sin35)

26.95(m) + 336.9 = 920 (2)

m = $\frac{920-336.9}{26.95}$

= 21.6 kg (1)

(if m/3 = 89.9kg maximum 3 marks)

(b) Calculate the reaction force the hinge exerts on the beam at Point C. (If you could not do (a), use m = 35.0 kg) (5 marks)

$\Sigma Fy = 0$ = $-W_{beam} - W_{mass} + T\sin30 + F_{RY}$ (½)

$F_{RY} = W_{beam} + W_{mass} - T\sin30$

= (21.6x9.8) +(25.0x9.8) – $\frac{1750}{3}$sin35 (1)

= 122 N

If Tension not T/3 F_{RY} = -547 N

$\Sigma Fx = 0$ = $T\cos30 + F_{Rx}$ (½)

$F_{RX} = -T\cos30$

= $\frac{1750}{3}$cos35 (1)

= 478 N

If tension not T/3 F_{Rx} = 1433 N

$F_R = \sqrt{122^2 + 478^2}$

= 493 N (½)

$\theta = tan^{-1}\left(\frac{122}{493}\right)$

= 14.3° (½)

F_R = 493 N Left @ 14.3° above horizontal (1)

If Tension no T/3, F_R = 1530 N 20.9° below

Question 4 **(11 marks)**

The International Space Station (ISS) orbits the Earth at an average altitude of 408 km. Due to factors such as tidal effects and aerodynamic drag from residual atmosphere, the ISS experiences an 'orbital decay' of approximately 2.05 km/month. The ISS has thrusters that periodically fire to return the ISS to its average altitude.

(a) Calculate the change in speed of the ISS from its average altitude in a time of 3.00 months. (5 marks)

$\Sigma F = F_c = F_g$ (½)

$$\frac{m_1 v^2}{r_0} = \frac{G m_1 m_2}{r_0^2}$$ (1)

$$v_i = \sqrt{\frac{GM}{r_i}}$$

$$= \sqrt{\frac{(6.67\times10^{-11})(5.97\times10^{24})}{(6.37\times10^{6}+408\times10^{3})}}$$ (1)

$= 7.665 \times 10^3$ ms^{-1}

$$v_f = \sqrt{\frac{GM}{r_f}}$$

$$= \sqrt{\frac{(6.67\times10^{-11})(5.97\times10^{24})}{(6.37\times10^{6}+408\times10^{3}-2.05\times10^{3}(3))}}$$ (1)

$= 7.668 \times 10^3$ ms^{-1}

$\Delta v = (7.665 - 7.68)\times10^3$ ms^{-1} (½)

$= 0.01 \times 10^3$ ms^{-1} (1.s.f) (1)

*If students find v_f and v_i from incorrect method: maximum 1.5 marks

(b) Explain the effect that this decay in orbit would have on the period of orbit. (3 marks)

- As radius decreases, velocity of ISS increases
- As $v = \frac{2\pi r}{T}$, $T \propto r/v$
- Hence, the period will decrease.

OR

- Kepler's 3rd Law states $\frac{r^3}{T^2} = k$
- $r^3 \propto T^2$
- hence, as r reduces, the period of orbit reduces.

(c) State the direction that the thrusters must apply the force to return the ISS to its average altitude and explain why. (3 marks)

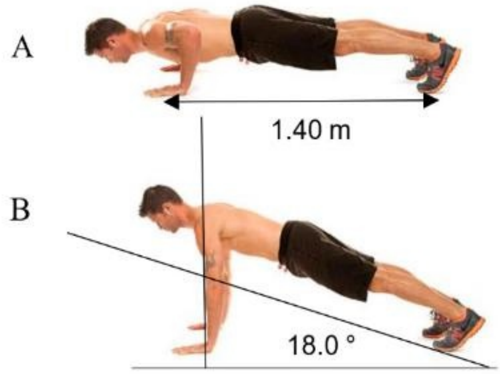
- Thruster must apply force in same direction (forward) or Thrusters "point backwards" ½ marks
- As $F_c = \frac{mv^2}{r}$, F_g providing F_c does not change substantially
- As v increases, r must increase to maintain ratio

OR

- Thrusters increase Ek of satellite
- which is converted into Ep, resulting in a higher orbit.

Question 5 **(9 marks)**

A student wants to know the effective force he applies from his hands when performing a push-up. He knows that his center of mass is located near his belly-button, a height of 1.10 m, and his mass is 71.0 kg.



(a) If the Student is in a 'low plank' as shown in diagram A, calculate the combined force that his hands must exert on the ground to remain stationary.

(3 marks)

$\Sigma\tau = 0 \quad \tau = rF\sin\theta$ (1)

$cwm = acwm$

Taking pivot about P

1.40(F) = 1.10(71.0x9.80) (1)

1.40 F = 765.4

F = 547 N down (1)

(b) Calculate the combined force of the ground exerted on the student's feet in this position.

(3 marks)

$\Sigma Fy = 0 = F_{Hands} - W + F_{Feet}$ (1)

F_{Feet} = (71x9.8) - 547 (1)

= 149 N upwards (1)

The student then performs a push-up and remains stationary in a 'high plank' as shown in diagram B. In doing so, his arms remain vertical and his body-line is now inclined to an angle of 18.0° above the horizontal.

(b) With the use of an appropriate equation, state and explain the change in combined force that his hands must exert on the ground, if any, that this position has compared to diagram B.

(3 marks)

$\Sigma\tau = 0 \quad \tau = rF\sin\theta$

$cwm = acwm$

$\theta = 90 - 18 = 72°$

$R_{\perp hands}(F_{Hands}) = R_{COM}(W_{\perp})$ (1)

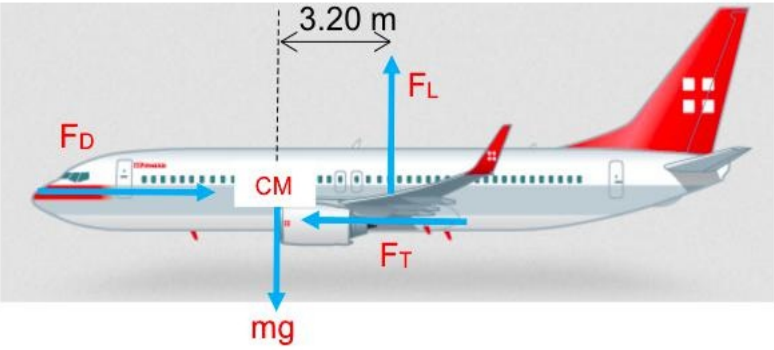
1.4 ~~sin72~~ F = 1.1 (mg) ~~sin72~~ (1)

No change on F_{hands} as both R⊥ and W⊥ are reduced by the same ratio

(1)

Question 6 **(6 marks)**

The forces acting on a 67,000 kg aircraft flying at a constant velocity are shown in the diagram. The engine thrust $F_T = 5.00 \times 10^5$ N acts on a line 1.60 m below the centre of mass CM. Calculate the drag force F_D and the distance above the centre of mass that it acts. Assume F_D and F_T are horizontal.



$\Sigma Fx = 0 = F_D + F_T$ (1)

$F_D = 5.00 \times 10^5$ N backwards (1)

$\Sigma Fy = 0 = F_{Lift} - W$

$F_{lift} = 67000(9.8)$
$= 6.57 \times 10^5$ N upwards
(not required for answer)

$\Sigma\tau = 0 \quad \tau = rF\sin\theta$
$cwm = acwm$ (1)
Taking pivot about CM

$rF_D + 1.60F_T = 3.20\,F_L$

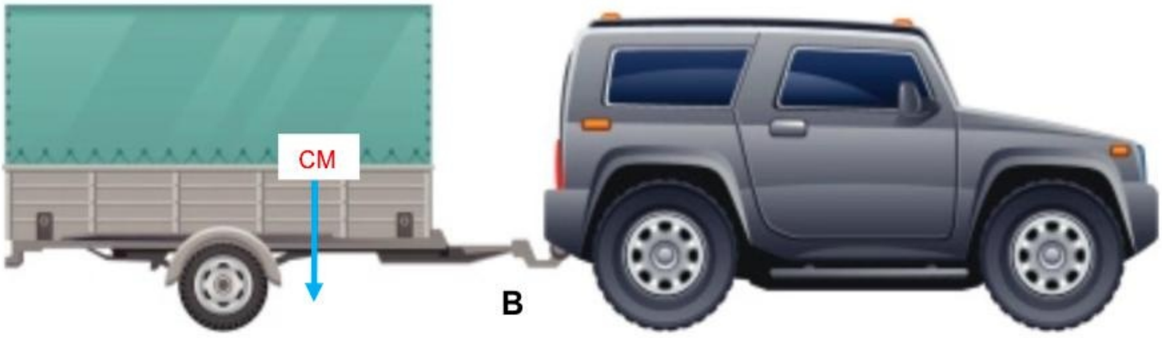
$r(5.00 \times 10^5) + 1.60(5.00 \times 10^5) = 3.20(67000 \times 9.80)$ (2)

$r(5.00 \times 10^5) = 1302400$

r = 2.60 m (1)

Question 7 **(6 marks)**

A 2250 kg trailer is attached to a stationary truck at point B. Calculate the normal force exerted by the road on the rear tyres at point A and the vertical force exerted on the trailer by the support B.



$\Sigma\tau = 0$ $\tau = rFsin\theta$
$cwm = acwm$ (1)
Taking pivot about A

1.1(2250x9.8) = (1.10+2.40) F_B (1)

24255 = 3.50 F_B

F_B = 6930 N (1)

$\Sigma Fy = 0 = F_A + F_B - W$ (1)

F_A = 2250(9.8) - 6930 (1)

= 1.51 x10^4 N upwards (1)

Question 8 **(5 marks)**

Calculate the altitude a satellite must be placed such that the magnitude of the gravitational field strength is half of that on the surface of the Earth.

$$g = \frac{GM_E}{r^2} = \frac{9.8}{2} = 4.90$$ (1)

$$4.9 = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(r_o)^2}$$ (1)

$r = 9.01 \times 10^6$ (1)

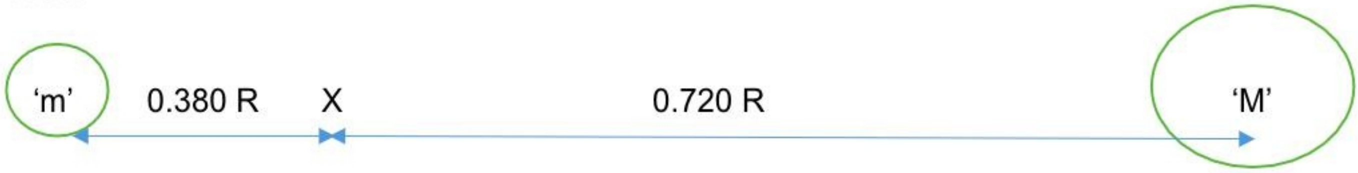
Alt = $r_o - r_e$ (1)

= $(9.01 – 6.37) \times 10^6$ m

= 2.64×10^6 m (1)

Question 9 **(4 marks)**

Consider a point, 'X' between two masses 'M' and 'm' where the net gravitational force produced by the two masses is zero. The diagram shows the relative distances in Rfrom each of the masses.



Calculate the ratio of the mases m/M given the information above.

$\Sigma F = 0 \quad F_m = F_M$ (1)

$F_m = F_M$

$$\frac{Gmm_m}{(0.38R)^2} = \frac{Gmm_M}{(0.720R)^2}$$

$$\frac{m_m}{(0.38R)^2} = \frac{m_M}{(0.720R)^2}$$ (1)

$$\frac{m_m}{m_M} = \frac{0.38^2}{0.72^2}$$ (1)

$= 0.279$ (1)

END OF TEST